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A plausible "s"-factor solution for many types of psychological and educational tests is one in which there is one general factor and "s - 1" group- or method-related factors. The bi-factor solution results from the constraint that each item has a non-zero loading on the primary dimension "alpha(sub j1)" and at most one of the "s - 1" group factors. This structure has been termed the "bi-factor" solution by K. J. Holzinger and F. Swineford (1937), but it also appears in the work of L. R. Tucker and K. G. Joreskog. All attempts at estimating the parameters of this model have been restricted to continuously measured variables; it has not been previously considered in the context of item response theory (IRT). It is conceivable that the bi-factor structure might arise in IRT-related problems. The purpose of this paper is to derive a bi-factor item response model for binary response data, and to develop a corresponding method of parameter estimation. This restriction leads to a major simplification of the likelihood equations that: (1) permits the statistical evaluation of problems of unlimited dimensionality; (2) permits conditional dependence among discrete and previously identified subsets of items; and (3) in some cases, provides more parsimonious factor solutions than an unrestricted full-information item factor analysis might provide. (Author/RLC)

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Full-Information Item Bi-Factor Analysis

ONR Technical Report

Robert D. Gibbons
University of Illinois at Chicago

Donald R. Hedeker
University of Illinois at Chicago

R. Darrell Bock
University of Chicago

July 1990

Dr. R.D. Gibbons
Biometric Laboratory
Illinois State Psychiatric Institute,
1601 W. Taylor St., Chicago, IL 60612, USA.

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The purpose of this paper is to derive a bi-factor item-response model for binary response data, and to develop a corresponding method of parameter estimation. This restriction leads to a major simplification of the likelihood equations that (1) permits the statistical evaluation of problems of unlimited dimensionality, (2) permits conditional dependence among discrete and previously identified subsets of items, and (3) in some cases provides more parsimonious factor solutions than an unrestricted full-information item factor analysis might provide (*e.g.*, Bock and Aitkin, 1981).

ABSTRACT

A plausible s-factor solution for many types of psychological and educational tests is one in which there is one general factor and $s-1$ group or method related factors. The bi-factor solution results from the constraint that each item has a non-zero loading on the primary dimension α_{j1} and at most one of the $s-1$ group factors. This structure has been termed the "bi-factor" solution by Holzinger & Swineford, but it also appears in the work of Tucker and Joreskog. All attempts at estimating the parameters of this model have been restricted to continuously measured variables; it has not been previously considered in the context of item-response theory (IRT). It is conceivable, however, that the bi-factor structure might arise in IRT related problems.

The purpose of this paper is to derive a bi-factor item-response model for binary response data, and to develop a corresponding method of parameter estimation. This restriction leads to a major simplification of the likelihood equations that (1) permits the statistical evaluation of problems of unlimited dimensionality, (2) permits conditional dependence among discrete and previously identified subsets of items, and (3) in some cases provides more parsimonious factor solutions than an unrestricted full-information item factor analysis might provide (*e.g.*, Bock and Aitkin, 1981).

1 Introduction

Consider the case in which, for n variables, an s-factor solution exists in which there is one general factor and $s-1$ group or method related factors. The bi-factor solution constrains each item to have a non-zero loading on the primary dimension α_{j1} and on not more than one of the $s-1$ group factors (*i.e.*, $\alpha_{jh}, h = 2, \ldots, s$). For four items, the factor-pattern matrix might be

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & 0 & \alpha_{33} \\ \alpha_{41} & 0 & \alpha_{43} \end{bmatrix}$$

This structure has been termed the "bi-factor" solution by Holzinger & Swineford (1937), inter-battery factor analysis by Tucker (1958), and is also one of the confirmatory factor analysis models considered by Joreskog (1969). In these applications, the model is restricted to test scores, assumed to be continuously distributed. It is easy, however to conceive of situations where the bi-factor pattern might arise at the item level. It is plausible for paragraph comprehension tests, for example, in which case the primary dimension describes the targeted aptitude and the additional factors describe knowledge of the content area within the paragraphs. In this context, items would be conditionally independent between paragraphs, but conditionally dependent within specific paragraphs.

The purpose of this paper is to derive an item-response model for binary response data that exhibit the bi-factor structure and to develop a corresponding method of parameter estimation. Of course, other types of tests that consist of items tapping different content areas would also be suitable for this type of analysis. As we will show, this restriction leads to a major simplification of the likelihood equations that (1) permits the statistical evaluation of problems of unlimited dimensionality, (2) permits conditional dependence among discrete and previously identified subsets of items, and (3) in some cases provides more parsimonious factor solutions than an unrestricted full-information item factor analysis might provide (*e.g.*, Bock and Aitkin, 1981). In the following sections, we derive the likelihood and its first derivatives so that an EM solution to item bi-factor analysis may be obtained.

2 Likelihood Evaluation

Stuart (1958) showed that if n variables follow a standardized multivariate normal distribution where the correlation $\rho_{ij} = \sum_{h=1}^{s} \alpha_{ih}\alpha_{jh}$ and α_{ih} is nonzero for

only one h, then the probability that the respective variables are simultaneously less than γ_j is,

$$P = \prod_{h=1}^{s} \int_{-\infty}^{\infty} \left[\prod_{j=1}^{n_h} F((\gamma_j - \alpha_{jh} y)/(1 - \alpha_{jh}^2)^{1/2}) \right] f(y) dy \tag{1}$$

where

$$f(t) = \exp(-\frac{1}{2} t^2)/(2\pi)^{1/2}$$

$$F(t) = \int_{-\infty}^{\gamma_j} f(t) dt$$

and n_h is the number of items loading on dimension h ($h = 1, \ldots, s$).

Equation (1) follows from the fact that if each variate is related to only a single dimension, then the s dimensions are independent, and the joint probability is simply the product of the s unidimensional probabilities. In the present context, this result only applies to the $s - 1$ "nuisance" dimensions (*i.e.*, $h = 2, \ldots, s$); if a primary dimension exists, it will not be independent of the other $s - 1$ dimensions. To compute this probability therefore requires a two-dimensional generalization of Stuart's (1958) original result.

To derive the two-dimensional result, we begin by noting that the probability of the primary dimension can be obtained using the formula of Dunnett and Sobel (1955),

$$P = \int_{-\infty}^{\infty} \left[\prod_{j=1}^{n} F((\gamma_j - \alpha_{j1} y)/(1 - \alpha_{j1}^2)^{1/2}) \right] f(y) dy, \tag{2}$$

which is valid as long as $\rho_{ij} = \alpha_i \alpha_j$. Of course, this directly implies a unidimensional problem. Combining the two results yields,

$$P = \int_{-\infty}^{\infty} \left\{ \prod_{h=2}^{s} \int_{-\infty}^{\infty} \left[\prod_{j=1}^{n_h} F\left(\frac{\gamma_j - \alpha_{j1} z - \alpha_{jh} y}{\sqrt{1 - \alpha_{j1}^2 - \alpha_{jh}^2}} \right) \right] f(y) dy \right\} f(z) dz, \tag{3}$$

which can be approximated to any practical degree of accuracy using Gauss-Hermite quadrature (Stroud and Sechrest, 1966). What is important about this result is, if the assumptions are reasonable (as they clearly are for many IRT applications), then the probability of any response pattern can be obtained by a two-dimensional integration, regardless of the dimensionality s.

For example, if $y_j = \sum_{h=1}^{s} \alpha_{jh} \theta_h + \varepsilon_j$ and we assume that

$$
\begin{aligned}
y_j &\sim N(0,1), \\
\boldsymbol{\theta} &\sim N(\mathbf{0},\mathbf{I}), \text{ and} \\
\varepsilon_j &\sim N(0, 1-\sum_{h=1}^{s}\alpha_{jh}),
\end{aligned}
$$

then the unconditional probability of observing score pattern $\mathbf{x} = \mathbf{x}_\ell$ is,

$$
P_\ell = \int_{-\infty}^{\infty}\left\{\prod_{h=2}^{s}\left[\int_{-\infty}^{\infty}\prod_{j=1}^{n_h}[F(\theta_1,\theta_h)]^{x_{\ell j}}[1-F(\theta_1,\theta_h)]^{1-x_{\ell j}}f(\theta_h)d\theta_h\right]\right\}f(\theta_1)d\theta_1, \tag{4}
$$

which can be approximated by,

$$
\hat{P}_\ell \cong \sum_{q_1}^{Q}\left\{\prod_{h=2}^{s}\left[\sum_{q_h}^{Q}\prod_{j=1}^{n_h}[F(X_{q_1},X_{q_h})]^{x_{\ell j}}[1-F(X_{q_1},X_{q_h})]^{1-x_{\ell j}}A(X_{q_h})\right]\right\}A(X_{q_1}), \tag{5}
$$

where

$$
F(X_{q_1},X_{q_h}) = F\left(-\frac{\gamma_j-\alpha_{j1}X_{q_1}-\alpha_{jh}X_{q_h}}{\sqrt{1-\alpha_{j1}^2-\alpha_{jh}^2}}\right),
$$

and $\mathbf{X}_q$ and $A(\mathbf{X}_q)$ are the nodes and corresponding weights of a Gauss-Hermite quadrature.

3 Marginal Maximum Likelihood Estimation

The parameters of the item bi-factor analysis model can be estimated by the method of marginal maximum likelihood using a variation of the approach described by Bock & Aitkin (1981). The parameters of this model include n "thresholds" or "intercepts", n primary factor loadings or "slopes" and a total of n factor loadings or slopes on the $h = 2,\ldots,s$ additional dimensions (*i.e.*, $\sum_{h=2}^{s} n_h = n$). The likelihood equations are derived as follows. Let

and

$$\bar{N}_h(\mathbf{X}) = \sum_{\ell=1}^{S} r_\ell [E_{\ell h}(X_{q_1})] L_{\ell h}(X_{q_1}, X_{q_h}) / P_\ell. \tag{13}$$

It should be noted that these equations are similar to those in the unrestricted case, except that in the bi-factor case, the conditional probability of response pattern $x_{\ell h}$ (*i.e.*, responses to items $j = 1, \ldots, n_h$ in subsection h for response pattern ℓ) is weighted by the factor, $E_{\ell h}(X_{q_1})$. Furthermore, since each item only appears in one subsection (h), the $\bar{N}$ now vary with h, in contrast to the unrestricted case. As such, the $\bar{N}_h$ denote the effective sample size for subset h at quadrature point (X_{q_1}, X_{q_h}). When weighted by $A(\mathbf{X})$ and summed over the quadrature nodes for each subsection, $\bar{N}_h$ yields the total number of respondents, whereas the corresponding weighting and summation for $\bar{r}_j$ yields the total number of respondents answering item j correctly.

From provisional parameter values, each E-Step yields $\bar{r}_j$ and $\bar{N}_h$, the expectations of the complete data statistics computed conditional on the incomplete data (see Bock, Gibbons, & Muraki, 1988). The subsequent M-step solves equation (10) using conventional maximum likelihood multiple probit analysis, substituting the provisional expectations of $\bar{r}_j$ and $\bar{N}_h$ (see Bock & Jones, 1968).

4 Illustration

To illustrate the application of the bi-factor IRT model, we have evaluated 20 items selected from an ACT natural science test, for a random sample of 1000 examinees (we are indebted to Terry Ackerman and Mark Reckase for these data). This test involves a series of questions regarding each of four paragraphs. For the purpose of this illustration, we selected the first 5 items from each of four paragraphs.

Table 1 displays the unrestricted promax-rotated 4-factor solution, which adequately fit these data (improvement in fit of a four-factor model over a three-factor model was $\chi^2_{17} = 31.59, p < .02$; the improvement in fit of five factors over four factors was not significant ($\chi^2_{16} = 18.44, p < .30$). Inspection of Table 1 reveals that each factor is dominated by items from a particular paragraph. In contrast, the estimated factor loadings for the bi-factor model (see Table 2) with $s = 5$ (*i.e.*, one primary dimension and four paragraph-specific dimensions) revealed a strong general ability dimension, as well as appreciable within paragraph associations. The fit of the restricted model was not significantly different from the fit of either the four-factor ($\chi^2_{45} = 23.83, p < .99$) or the five-factor ($\chi^2_{60} = 43.22, p < .95$) unrestricted models. Inspection

$$
\begin{aligned}
P_\ell &= P(\mathbf{x} = \mathbf{x}_\ell) \\
&= \int_{\theta_1} \left\{ \prod_{h=2}^{s} \int_{\theta_h} \prod_{j=1}^{n_h} [F_j(\boldsymbol{\theta})]^{x_{\ell j}} [1 - F_j(\boldsymbol{\theta})]^{1 - x_{\ell j}} f(\theta_h) d\theta_h \right\} f(\theta_1) d\theta_1 \\
&= \int_{\theta_1} \left\{ \prod_{h=2}^{s} \int_{\theta_h} L_{\ell h}(\theta) f(\theta_h) d\theta_h \right\} f(\theta_1) d\theta_1. \qquad (6)
\end{aligned}
$$

Then the log likelihood is,

$$
\log L = \sum_{\ell=1}^{S} r_\ell \log \hat{P}_\ell \qquad (7)
$$

where S denotes the number of unique response patterns. The derivative of the log marginal likelihood with respect to a general item parameter ν_j is as follows.
Let

$$
E_{\ell h}(\theta_1) = \frac{\left[\prod_{h=2}^{s} \int_{\theta_h} L_{\ell h}(\theta) f(\theta_h) d\theta_h\right]}{\int_{\theta_h} L_{\ell h}(\theta) f(\theta_h) d\theta_h}. \qquad (8)
$$

Then

$$
\frac{\partial \log L}{\partial \nu_j} = \sum_{\ell}^{S} \frac{r_\ell}{P_\ell} \left(\frac{\partial P_\ell}{\partial \nu_j} \right) \qquad (9)
$$

$$
= \sum_{\ell=1}^{S} \frac{r_\ell}{P_\ell} \int_{\theta_1} E_{\ell h}(\theta_1) \left\{ \int_{\theta_h} \left(\frac{x_{\ell j} - F_j(\boldsymbol{\theta})}{F_j(\boldsymbol{\theta})[1 - F_j(\boldsymbol{\theta})]} \right) L_{\ell h}(\boldsymbol{\theta}) \frac{\partial F_j(\boldsymbol{\theta})}{\partial \nu_j} f(\theta_h) d\theta_h \right\} f(\theta_1) d\theta_1. \qquad (10)
$$

Following Bock and Aitkin (1981), the marginal likelihood equations can be solved, using the EM algorithm of Dempster, Laird & Rubin (1977), by replacing the integrals with Gauss-Hermite quadratures and rearranging terms into the two-dimensional form:

$$
\sum_{q_1}^{Q} \sum_{q_h}^{Q} \frac{\bar{r}_j(\mathbf{X}) - \bar{N}_s(\mathbf{X}) F_j(\mathbf{X})}{F_j(\mathbf{X})[1 - F_j(\mathbf{X})]} \left(\frac{\partial F_j(\mathbf{X})}{\partial \nu_j} \right) A(X_{q_h}) A(X_{q_1}), \qquad (11)
$$

where

$$
\bar{r}_j(\mathbf{X}) = \sum_{\ell=1}^{S} r_\ell x_{\ell j} [E_{\ell h}(X_{q_1})] L_{\ell h}(X_{q_1}, X_{q_h}) / P_\ell \qquad (12)
$$

of the loadings within each paragraph reveals that the intra-paragraph item associations are quite variable.

As a computational note, we should point out that the numerical precision of the bi-factor solution represents a major improvement over the unrestricted solution. Given that the bi-factor solution only requires approximation of a two-dimensional integral, we were able to use 100 quadrature points (*i.e.*, 10 in each dimension) instead of the 243 quadrature points used in the unrestricted five factor solution, (*i.e.*, 3 in each dimension). Five factors probably represents the highest dimensional solution that is computational tractable at this time. Parameters of the unrestricted models were estimated using the TESTFACT program (Wilson, Wood & Gibbons, 1984).

5 A Simple Structure Model

Consider an orthogonal simple structure factor model in which each item loads on one and only one of s dimensions. This satisfies a complete simple structure model as defined by Thurstone (1947), which for measurement data could be evaluated using methods for confirmatory factor analysis (Joreskog, 1969). This is, of course, a simplification of the bi-factor model in which there is no primary dimension. In this case, the unconditional probability in (5) is reduced to the unidimensional form,

$$P_\ell \cong \prod_{h=1}^{s} \left[\sum_{q_h}^{Q} \left\{ \prod_{j=1}^{n_h} [F(X_{q_h})]^{x_{\ell j}} [1 - F(X_{q_h})]^{1-x_{\ell j}} \right\} A(X_{q_h}) \right], \tag{14}$$

where

$$F(X_{q_h}) = F\left(-\frac{\gamma_j - \alpha_{jh} X_{q_h}}{\sqrt{1 - \alpha_{jh}^2}} \right);$$

that is, (5) reduces to the product of the s independent unidimensional probabilities. The likelihood equations in (11) can then be approximated by,

$$\frac{\partial \log L}{\partial \nu_j} \cong \sum_{q_h}^{Q} \frac{\bar{r}_j(X_{q_h}) - \bar{N}_h(X_{q_h}) F_j(X_{q_h})}{F_j(X_{q_h})[1 - F_j(X_{q_h})]} \left(\frac{\partial F_j(X_{q_h})}{\partial \nu_j} \right) A(X_{q_h}), \tag{15}$$

where

$$\bar{r}_j(X_{q_h}) = \sum_{\ell=1}^{S} r_\ell x_{\ell j} L_{\ell h}(X_{q_h}) / e_h \tag{16}$$

and

$$\tilde{N}_h(X_{q_h}) = \sum_{\ell=1}^{S} r_\ell L_{\ell h}(X_{q_h})/e_h. \tag{17}$$

In this case, e_h represents the constant

$$e_h = \sum_{q_h}^{Q} L_{\ell h}(X_{q_h}) A(X_{q_h}),$$

and

$$P_\ell = \prod_{h=1}^{s} e_h$$

It is interesting to note that $\tilde{r}_j$ and $\tilde{N}_h$ now only contain information from the specific subset of items (h) for which item j is a member. This is, of course, due to the independence between the subsets that results from the simple structure.

Application of the simple structure model to the ACT natural science test example yields the item-parameters displayed in Table 3. Inspection of the parameter estimates in Table 3 reveals that removal of the primary factor increases the magnitude of the loadings on the individual paragraph dimensions. In terms of model fit, both the bi-factor model ($\chi^2_{20} = 336, p < .0001$) and the unrestricted four-factor model ($\chi^2_{65} = 361, p < .0001$) provide significant improvements in fit over the simple structure model, indicating that the test is in fact measuring a primary ability dimension and not merely four independent realms of knowledge.

6 Discussion

The bi-factor model presented here provides a natural alternative to the traditional conditionally-independent unidimensional IRT model. When potential sources of conditional dependence are known in advance, as in the case of paragraph comprehension tests or tests in which two or more methods of item presentation are involved, the item bi-factor solution provides an excellent alternative. An attractive by-product of this model is that it requires only the evaluation of a two-dimensional integral, regardless of the number of potential subtests, paragraphs, or content areas. These different content areas are, of course, assumed to be independent conditional on the primary ability dimension that the test was designed to measure. As such, the limitations on the dimensionality of the full-information item factor analysis model embodied in

the TESTFACT program (Wilson, Wood & Gibbons, 1984), do not apply. Of course, the subsections (*e.g.*, paragraphs) must be known in advance.

In certain situations, for example psychiatric measurement (Gibbons, 1985), the existence of a primary dimension (*e.g.*, depression), is itself at question. In this case, comparison of the bi-factor and simple factor solutions presented here is of particular interest. Item bi-factor analysis could therefore help answer the question of whether depression is a unitary disorder or a mixture of a series of qualitatively distinct abnormalities; a question that has long plagued psychiatric researchers. Comparison of the fit of the bi-factor and simple structure models provides a tool for investigating such problems in psychiatric research and other areas as well.

Finally, those cases in which little is known about the structure of a particular test, but little confidence can be placed in the assumption of conditional independence, the more general solution presented by Gibbons *et. al.* (1989), using Clark's (1961) formulae for the moments of n jointly normal variables, could be used. This procedure uses a direct approximation to the multivariate normal distribution that underlies the item-response function, without restrictions on the form of the inter-item residual covariances. With it, the assumption of conditional independence is not required. Further work in this area is underway.

References

Bock, R. D., & Aitkin, M. (1981). Marginal maximum likelihood estimation of item parameters: application of an EM algorithm. *Psychometrika*, **46**, 443-459.

Bock, R. D., Gibbons, R. D., & Muraki, E. (1988). Full-information item factor analysis. em Applied Psychological Measurement, **12**, 261-280.

Bock, R. D., & Jones, L. V. (1968). *The measurement and prediction of judgment and choice*. San Fransisco: Holden-Day.

Clark, C. E. (1961). The greatest of a finite set of random variables. *Operations Research*, **9**, 145-162.

Dempster, A. P., Laird, N. M., & Rubin, D. B. (1977). Maximum likelihood from incomplete data via the EM algorithm (with discussion). *Journal of the Royal Statistical Society, Series B*, **39**, 1-38.

Dunnett, C. W. & Sobel, M. (1955). Approximations to the probability integral and certain percentage points of a multivariate analogue of Student's *t*-distribution. *Biometrika*, **42**, 258 - 260.

Gibbons, R. D., Bock, R. D. & Hedeker, D. R. (1989). Conditional Dependence. *ONR Final Report*, No. 89-1.

Gibbons R.D., Clark D.C., Cavanaugh S.V. (1985). Application of modern psychometric theory in psychiatric research. *Journal of of Psychiatric Research*, *19*, 43-55.

Holzinger, K. J., & Swineford, F. (1937). The bi-factor method. *Psychometrika*, **2**, 41-54.

Joreskog, K. G. (1969). A general approach to confirmatory maximum likelihood factor analysis. *Psychometrika*, **34**, 183-202.

Stroud, A. H., & Sechrest, D. (1966). *Gaussian Quadrature Formulas*. Englewood Cliffs, NJ: Prentice Hall.

Stuart, A. (1958). Equally correlated variates and the multinormal integral. *J. Roy. Statist. Soc. Ser. B*, **20**, 373-378.

Thurstone, L. L. *Multiple Factor Analysis*. Chicago: University of Chicago Press, 1947.

Tucker, L. R. (1958). An inter-battery method of factor analysis. *Psychometrika*, **23**, 111-136.

Wilson, D., Wood, R., & Gibbons, R. D. (1984). *TESTFACT: Test scoring, item statistics, and item factor analysis.* Mooresville, IN.: Scientific Software, Inc.

Table 1

Full-Information Item Factor Analysis - Unrestricted Promax Solution
ACT Natural Science Test - 20 items and 1000 subjects

Item	γ_j	α_{j1}	α_{j2}	α_{j3}	α_{j4}
1	-.215	**.401**	-.005	-.036	.216
2	-.385	**.185**	-.019	-.007	.105
3	-.356	**.667**	-.070	-.081	-.081
4	-.098	**.619**	.013	.044	-.022
5	-.029	**.562**	-.092	-.059	.119
6	-.582	.129	.068	**.256**	.030
7	-.585	.184	-.211	**.419**	.102
8	-.137	-.037	-.061	**.025**	.172
9	-.246	.238	.063	**.362**	-.284
10	-.089	-.224	.128	**.620**	.060
11	-.049	.182	.135	-.034	**.311**
12	-.407	-.024	-.065	.124	**.320**
13	-.265	.247	.082	.020	**.173**
14	-.051	.137	.005	.007	**.585**
15	.040	.224	.129	-.045	**.295**
16	.345	.153	**.289**	-.122	-.109
17	.167	-.007	**.682**	.089	-.044
18	.172	-.096	**.520**	-.024	.120
19	.543	.008	**.500**	.067	.091
20	.672	-.073	**-.010**	.004	.163

Table 2

Full-Information Item Bi-Factor Analysis
ACT Natural Science Test - 20 items and 1000 subjects

Item	γ_j	α_{j1}	α_{j2}	α_{j3}	α_{j4}	α_{j5}
1	-.230	.524	.129			
2	-.392	.232	.115			
3	-.370	.411	.427			
4	-.118	.548	.278			
5	-.046	.489	.338			
6	-.593	.311		.277		
7	-.600	.376		.314		
8	-.138	.087		-.019		
9	-.259	.207		.390		
10	-.103	.226		.476		
11	-.062	.484			.141	
12	-.413	.261			.135	
13	-.277	.423			.199	
14	-.066	.573			.187	
15	.025	.492			.260	
16	.340	.112				.261
17	.150	.306				.662
18	.160	.240				.571
19	.528	.340				.493
20	.671	.061				.031

Table 3

Full-Information Simple Structure Item Factor Analysis
ACT Natural Science Test - 20 items and 1000 subjects

Item	γ_j	α_{j1}	α_{j2}	α_{j3}	α_{j4}
1	-.224	.482			
2	-.391	.251			
3	-.368	.571			
4	-.111	.612			
5	-.040	.585			
6	-.592		.408		
7	-.597		.467		
8	-.138		.032		
9	-.258		.429		
10	-.102		.509		
11	-.056			.489	
12	-.412			.297	
13	-.273			.449	
14	-.058			.591	
15	.031			.566	
16	.341				.282
17	.157				.732
18	.163				.616
19	.534				.597
20	.671				.057

Dr. Terry Ackerman
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. James Algina
1403 Norman Hall
University of Florida
Gainesville, FL 32605

Dr. Erling B. Andersen
Department of Statistics
Studiestraede 6
1455 Copenhagen
DENMARK

Dr. Ronald Armstrong
Rutgers University
Graduate School of Management
Newark, NJ 07102

Dr. Eva L. Baker
UCLA Center for the Study
of Evaluation
145 Moore Hall
University of California
Los Angeles, CA 90024

Dr. Laura L. Barnes
College of Education
University of Toledo
2801 W. Bancroft Street
Toledo, OH 43606

Dr. William M. Bart
University of Minnesota
Dept. of Educ. Psychology
330 Burton Hall
178 Pillsbury Dr., S.E.
Minneapolis, MN 55455

Dr. Isaac Bejar
Mail Stop: 10-R
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Dr. Menucha Birenbaum
School of Education
Tel Aviv University
Ramat Aviv 69978
ISRAEL

Dr. Arthur S. Blaiwes
Code N712
Naval Training Systems Center
Orlando, FL 32813-7100

Dr. Bruce Blomme
Defense Manpower Data Center
99 Pacific St.
Suite 155A
Monterey, CA 93943-3231

Cdt. Arnold Bohrer
Sectie Psychologisch Onderzoek
Rekruterings-En Selectiecentrum
Kwartier Koningen Astrid
Bruynstraat
1120 Brussels, BELGIUM

Dr. Robert Breaux
Code 281
Naval Training Systems Center
Orlando, FL 32826-3224

Dr. Robert Brennan
American College Testing
Programs
P. O. Box 168
Iowa City, IA 52243

Dr. Gregory Candell
CTB/McGraw-Hill
2500 Garden Road
Monterey, CA 93940

Dr. John B. Carroll
409 Elliott Rd., North
Chapel Hill, NC 27514

Dr. John M. Carroll
IBM Watson Research Center
User Interface Institute
P.O. Box 704
Yorktown Heights, NY 10598

Dr. Robert M. Carroll
Chief of Naval Operations
OP-01B2
Washington, DC 20350

Dr. Raymond E. Christal
UES LAMP Science Advisor
AFHRL/MOEL
Brooks AFB, TX 78235

Mr. Hua Hua Chung
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Dr. Norman Cliff
Department of Psychology
Univ. of So. California
Los Angeles, CA 90089-1061

Director, Manpower Program
Center for Naval Analyses
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Director,
Manpower Support and
Readiness Program
Center for Naval Analysis
2000 North Beauregard Street
Alexandria, VA 22311

Dr. Stanley Collyer
Office of Naval Technology
Code 222
800 N. Quincy Street
Arlington, VA 22217-5000

Dr. Hans F. Crombag
Faculty of Law
University of Limburg
P.O. Box 616
Maastricht
The NETHERLANDS 6200 MD

Ms. Carolyn R. Crone
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

Dr. Timothy Davey
American College Testing Program
P.O. Box 168
Iowa City, IA 52243

Dr. C. M. Dayton
Department of Measurement
Statistics & Evaluation
College of Education
University of Maryland
College Park, MD 20742

Dr. Ralph J. DeAyala
Measurement, Statistics,
and Evaluation
Benjamin Bldg., Rm. 4112
University of Maryland
College Park, MD 20742

Dr. Lou DeBello
CERL
University of Illinois
103 South Mathews Avenue
Urbana, IL 61801

Dr. Dattprasad Divgi
Center for Naval Analyses
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Mr. Hei-Ki Dong
Bell Communications Research
Room PYA-1K207
P.O. Box 1320
Piscataway, NJ 08855-1320

Dr. Fritz Drasgow
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Dr. Stephen Dunbar
224B Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. James A. Earles
Air Force Human Resources Lab
Brooks AFB, TX 78235

Dr. Susan Embretson
University of Kansas
Psychology Department
426 Fraser
Lawrence, KS 66045

Dr. George Englehard, Jr.
Division of Educational Studies
Emory University
210 Fishburne Bldg.
Atlanta, GA 30322

Dr. Benjamin A. Fairbank
Operational Technologies Corp.
5825 Callaghan, Suite 225
San Antonio, TX 78228

Dr. P.A. Federico
Code 51
NPRDC
San Diego, CA 92152-6800

Dr. Leonard Feldt
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Richard L. Ferguson
American College Testing
P.O. Box 168
Iowa City, IA 52243

Dr. Gerhard Fischer
Liebiggasse 5/3
A 1010 Vienna
AUSTRIA

Dr. Myron Fischl
U.S. Army Headquarters
DAPE-MRR
The Pentagon
Washington, DC 20310-0300

Prof. Donald Fitzgerald
University of New England
Department of Psychology
Armidale, New South Wales 2351
AUSTRALIA

Mr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Alfred R. Fregly
AFOSR/NL, Bldg. 410
Bolling AFB, DC 20332-6448

Dr. Robert D. Gibbons
Illinois State Psychiatric Inst.
Rm 529W
1601 W. Taylor Street
Chicago, IL 60612

Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01003

Dr. Drew Gitomer
Educational Testing Service
Princeton, NJ 08541

Dr. Robert Glaser
Learning Research
& Development Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

Michael Habon
DORNIER GMBH
P.O. Box 1420
D-7990 Friedrichshafen 1
WEST GERMANY

Prof. Edward Haertel
School of Education
Stanford University
Stanford, CA 94305

Dr. Ronald K. Hambleton
University of Massachusetts
Laboratory of Psychometric
and Evaluative Research
Hills South, Room 152
Amherst, MA 01003

Dr. Delwyn Harnisch
University of Illinois
51 Gerty Drive
Champaign, IL 61820

Dr. Grant Henning
Senior Research Scientist
Division of Measurement
Research and Services
Educational Testing Service
Princeton, NJ 08541

Ms. Rebecca Hetter
Navy Personnel R&D Center
Code 63
San Diego, CA 92152-6800

Dr. Thomas M. Hirsch
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Paul W. Holland
Educational Testing Service, 21-T
Rosedale Road
Princeton, NJ 08541

Dr. Paul Horst
677 G Street, #184
Chula Vista, CA 92010

Dr. Lloyd Humphreys
University of Illinois
Department of Psychology
603 East Daniel Street
Champaign, IL 61820

Dr. Steven Hunka
3-104 Educ. N.
University of Alberta
Edmonton, Alberta
CANADA T6G 2G5

Dr. Huynh Huynh
College of Education
Univ. of South Carolina
Columbia, SC 29208

Dr. Robert Jannarone
Elec. and Computer Eng. Dept.
University of South Carolina
Columbia, SC 29208

Dr. Kumar Joag-dev
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright Street
Champaign, IL 61820

Dr. Douglas H. Jones
1280 Woodfern Court
Toms River, NJ 08753

Dr. Brian Junker
Carnegie-Mellon University
Department of Statistics
Schenley Park
Pittsburgh, PA 15213

Dr. Milton S. Katz
European Science Coordination
Office
U.S. Army Research Institute
Box 65
FPO New York 09510-1500

Prof. John A. Keats
Department of Psychology
University of Newcastle
N.S.W. 2308
AUSTRALIA

Dr. Jwa-keun Kim
Department of Psychology
Middle Tennessee State
University
P.O. Box 522
Murfreesboro, TN 37132

Mr. Soon-Hoon Kim
Computer-based Education
Research Laboratory
University of Illinois
Urbana, IL 61801

Dr. G. Gage Kingsbury
Portland Public Schools
Research and Evaluation Department
501 North Dixon Street
P. O. Box 3107
Portland, OR 97209-3107

Dr. William Koch
Box 7246, Meas. and Eval. Ctr.
University of Texas-Austin
Austin, TX 78703

Dr. Richard J. Koubek
Department of Biomedical
& Human Factors
139 Engineering & Math Bldg.
Wright State University
Dayton, OH 45435

Dr. Leonard Kroeker
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Dr. Jerry Lehnus
Defense Manpower Data Center
Suite 400
1600 Wilson Blvd
Rosslyn, VA 22209

Dr. Thomas Leonard
University of Wisconsin
Department of Statistics
1210 West Dayton Street
Madison, WI 53705

Dr. Michael Levine
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. Charles Lewis
Educational Testing Service
Princeton, NJ 08541-0001

Mr. Rodney Lim
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Dr. Robert L. Linn
Campus Box 249
University of Colorado
Boulder, CO 80309-0249

Dr. Robert Lockman
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Frederic M. Lord
Educational Testing Service
Princeton, NJ 08541

Dr. Richard Luecht
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. George B. Macready
Department of Measurement
Statistics & Evaluation
College of Education
University of Maryland
College Park, MD 20742

Dr. Gary Marco
Stop 31-E
Educational Testing Service
Princeton, NJ 08451

Dr. Clessen J. Martin
Office of Chief of Naval
Operations (OP 13 F)
Navy Annex, Room 2832
Washington, DC 20350

Dr. James R. McBride
The Psychological Corporation
1250 Sixth Avenue
San Diego, CA 92101

Dr. Clarence C. McCormick
HQ, USMEPCOM/MEPCT
2500 Green Bay Road
North Chicago, IL 60064

Mr. Christopher McCusker
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Dr. Robert McKinley
Educational Testing Service
Princeton, NJ 08541

Mr. Alan Mead
c/o Dr. Michael Levine
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. Timothy Miller
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Robert Mislevy
Educational Testing Service
Princeton, NJ 08541

Dr. William Montague
NPRDC Code 13
San Diego, CA 92152-6800

Ms. Kathleen Moreno
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Headquarters Marine Corps
Code MPI-20
Washington, DC 20380

Dr. Ratna Nandakumar
Educational Studies
Willard Hall, Room 213E
University of Delaware
Newark, DE 19716

Dr. Harold F. O'Neil, Jr.
School of Education - WPH 801
Department of Educational
Psychology & Technology
University of Southern California
Los Angeles, CA 90089-0031

Dr. James B. Olsen
WICAT Systems
1875 South State Street
Orem, UT 84058

Dr. Judith Orasanu
Basic Research Office
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Jesse Orlansky
Institute for Defense Analyses
1801 N. Beauregard St.
Alexandria, VA 22311

Dr. Peter J. Pashley
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Wayne M. Patience
American Council on Education
GED Testing Service, Suite 20
One Dupont Circle, NW
Washington, DC 20036

Dr. James Paulson
Department of Psychology
Portland State University
P.O. Box 751
Portland, OR 97207

Dr. Mark D. Reckase
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Malcolm Ree
AFHRL/MOA
Brooks AFB, TX 78235

Mr. Steve Reise
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455-0344

Dr. Carl Ross
CNET-PDCD
Building 90
Great Lakes NTC, IL 60088

Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

Dr. Fumiko Samejima
Department of Psychology
University of Tennessee
310B Austin Peay Bldg.
Knoxville, TN 37916-0900

Mr. Drew Sands
NPRDC Code 62
San Diego, CA 92152-6800

Lowell Schoer
Psychological & Quantitative
Foundations
College of Education
University of Iowa
Iowa City, IA 52242

Dr. Mary Schratz
905 Orchid Way
Carlsbad, CA 92009

Dr. Dan Segall
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Robin Shealy
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Dr. Kazuo Shigemasu
7-9-24 Kugenuma-Kaigan
Fujisawa 251
JAPAN

Dr. Richard E. Snow
School of Education
Stanford University
Stanford, CA 94305

Dr. Richard C. Sorensen
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Judy Spray
ACT
P.O. Box 168
Iowa City, IA 52243

Dr. Martha Stocking
Educational Testing Service
Princeton, NJ 08541

Dr. Peter Stoloff
Center for Naval Analyses
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. William Stout
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Dr. Hariharan Swaminathan
Laboratory of Psychometric and
Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

Mr. Brad Sympson
Navy Personnel R&D Center
Code-62
San Diego, CA 92152-6800

Dr. John Tangney
AFOSR/NL, Bldg. 410
Bolling AFB, DC 20332-6448

Dr. Kikumi Tatsuoka
Educational Testing Service
Mail Stop 03-T
Princeton, NJ 08541

Dr. Maurice Tatsuoka
Educational Testing Service
Mail Stop 03-T
Princeton, NJ 08541

Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044

Mr. Thomas J. Thomas
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

Mr. Gary Thomasson
University of Illinois
Educational Psychology
Champaign, IL 61820

Dr. Robert Tsutakawa
University of Missouri
Department of Statistics
222 Math. Sciences Bldg.
Columbia, MO 65211

Dr. Ledyard Tucker
University of Illinois
Department of Psychology
603 E. Daniel Street
Champaign, IL 61820

Dr. David Vale
Assessment Systems Corp.
2233 University Avenue
Suite 440
St. Paul, MN 55114

Dr. Frank L. Viono
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Howard Wainer
Educational Testing Service
Princeton, NJ 08541

Dr. Michael T. Waller
University of Wisconsin-Milwaukee
Educational Psychology Department
Box 413
Milwaukee, WI 53201

Dr. Ming-Mei Wang
Educational Testing Service
Mail Stop 03-T
Princeton, NJ 08541

Dr. Thomas A. Warm
FAA Academy AAC934D
P.O. Box 25082
Oklahoma City, OK 73125

Dr. Brian Waters
HumRRO
1100 S. Washington
Alexandria, VA 22314

Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455-0344

Dr. Ronald A. Weitzman
Box 146
Carmel, CA 93921

Major John Welsh
AFHRL/MOAN
Brooks AFB, TX 78223

Dr. Douglas Wetzel
Code 51
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Rand R. Wilcox
University of Southern
California
Department of Psychology
Los Angeles, CA 90089-1061

German Military Representative
ATTN: Wolfgang Wildgrube
Streitkraefteamt
D-5300 Bonn 2
4000 Brandywine Street, NW
Washington, DC 20016

Dr. Bruce Williams
Department of Educational
Psychology
University of Illinois
Urbana, IL 61801

Dr. Hilda Wing
Federal Aviation Administration
900 Independence Ave, SW
Washington, DC 20591

Mr. John H. Wolfe
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. George Wong
Biostatistics Laboratory
Memorial Sloan-Kettering
Cancer Center
1275 York Avenue
New York, NY 10021

Dr. Wallace Wulfeck, III
Navy Personnel R&D Center
Code 51
San Diego, CA 92152-6800

Dr. Kentaro Yamamoto
02-T
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Dr. Wendy Yen
CTB/McGraw Hill
Del Monte Research Park
Monterey, CA 93940

Dr. Joseph L. Young
National Science Foundation
Room 320
1800 G Street, N.W.
Washington, DC 20550

Mr. Anthony R. Zara
National Council of State
Boards of Nursing, Inc.
625 North Michigan Avenue
Suite 1544
Chicago, IL 60611